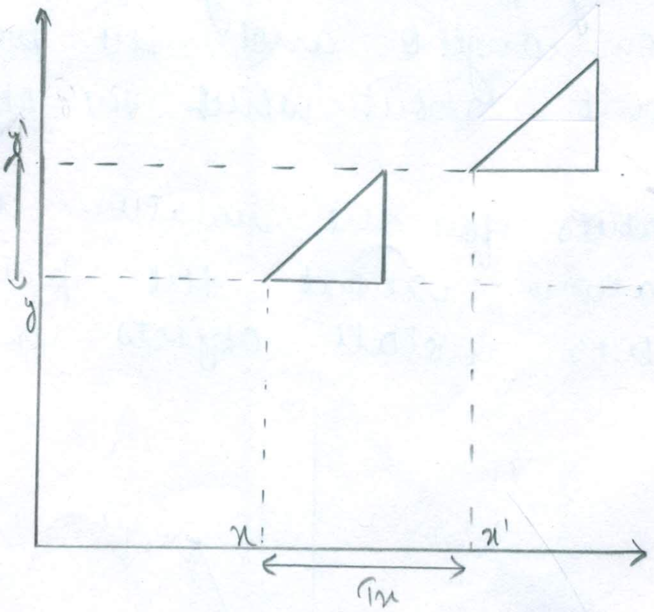
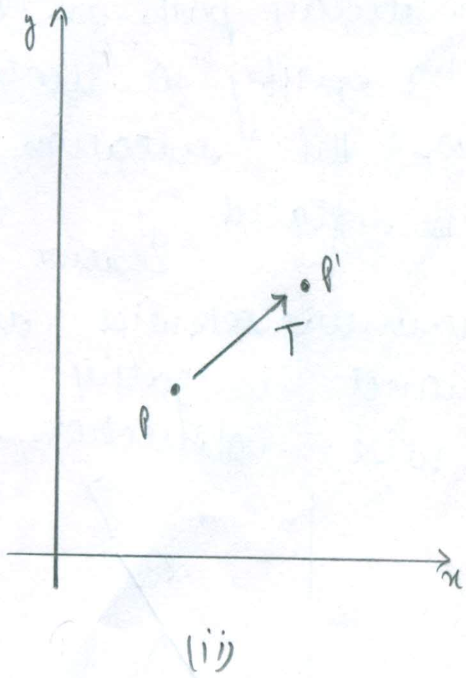
Transformation :→ Transformation means changing some graphics into something else by applying rules. We can have various types of transformations. And animations are produced by moving the objects in a scene along animation paths. Changes in orientation, size, and shape are accomplished with Geometric transformation that alter the co-ordinate description of objects.

The Basic geometric transformations are Translation, Rotation and Scaling.

I Translation :→ A translation is applied to an object by repositioning it along a straight-line path from one co-ordinate location to another. We translate a two-dimensional point by adding translation distances t_x and t_y to the original position (x, y) to move the point to a new position (x', y')



(i)



(ii)

$$x' = x + t_x$$

$$y' = y + t_y$$

In diagrams (ii) and (i) we translating a point from position P to position P' with translation vector T. The translation distance pair (t_x, t_y) is called a translation vector or Shift vector.

We can express the translation equation $x' = x + t_x$, $y' = y + t_y$ as a single matrix equation by using column vectors to represent co-ordinate positions and the translation vector.

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

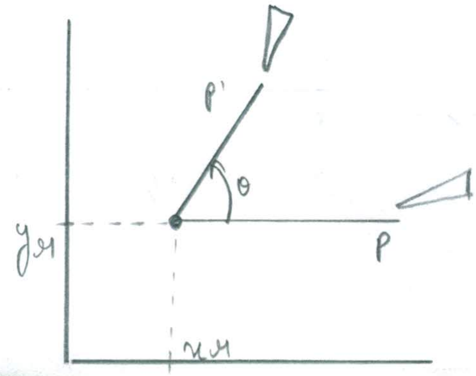
$$P' = P + T$$

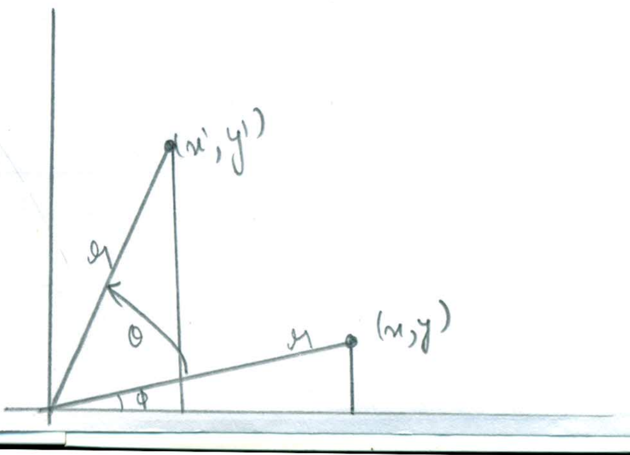
and another row wise.

$$P = [x, y] \quad P' = [x', y'] \quad T = [t_x, t_y]$$

II <u>ROTATION</u>:- A two-dimensional rotation is applied to an object by repositioning it along a circular path in the xy plane. To generate a rotation we specify a rotation angle θ and the position (x_r, y_r) of the rotation point about which the object is to be rotated.

Positive values for the rotation angle define counterclockwise rotations about the pivot point and negative values rotate objects in the clock-wise direction.





Using standard trigonometric identities, we can express the transformed coordinates in terms of angles θ and ϕ as 3-2

$$① \begin{cases} x' = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ y' = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{cases}$$

The original co-ordinates of the point in polar coordinates are :-

$$② \begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$$

Substituting expression ① and ② we obtain the transformation equations for rotating a point at position (x, y) through an angle θ about the origin.

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

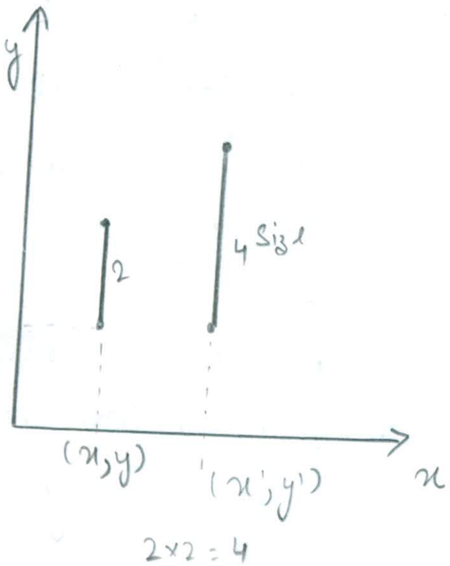
$$P' = R.P$$

where rotation matrix is:-

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

III **SCALING** :→ A scaling transformation alters the size of an object. This operation can be carried out for polygons by multiplying the co-ordinate values (x, y) of each vertex by Scaling Factors S_x and S_y to produce the transformed co-ordinates (x', y')

$$x' = x \cdot S_x \quad , \quad y' = y \cdot S_y$$



Scaling factor S_x scales objects in the x direction, while S_y scales in the y direction. The transformation equations can also be written in the matrix form.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = S \cdot P$$

where S is the 2 by 2 scaling matrix in equation

Matrix Representation & Homogeneous Co-ordinates

Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = T(t_x, t_y) \cdot P$$

Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = R(\theta) \cdot P$$

Scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = S(S_x, S_y) \cdot P$$

Composite Transformation

Matrix product of individual transformation →

Translation → for eg two translation vector (tx_1, ty_1) and (tx_2, ty_2)

$$P' = T(tx_2, ty_2) \cdot \{T(tx_1, ty_1) \cdot P\}$$
$$= \{T(tx_2, ty_2) \cdot T(tx_1, ty_1)\} \cdot P$$

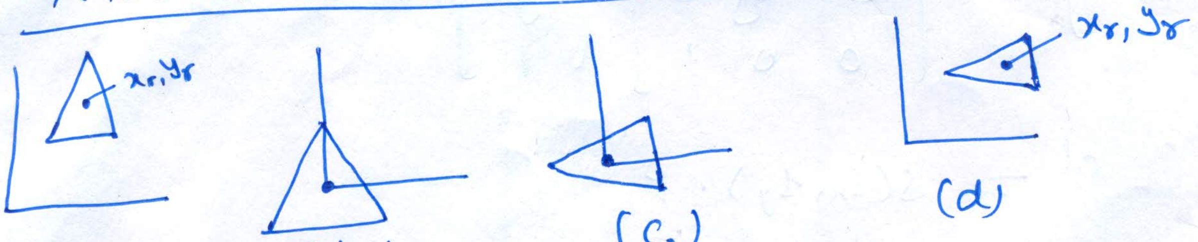
$$\begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx_1 + tx_2 \\ 0 & 1 & ty_1 + ty_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation $\quad P' = R(\theta_1 + \theta_2) \cdot P$

Scaling

$$S(Sx_2, Sy_2) \cdot S(Sx_1, Sy_1) = S(Sx_1 \cdot Sx_2, Sy_1 \cdot Sy_2)$$

General Pivot Point Rotation →



(a) (b) (c) (d)

translate – rotate – translate

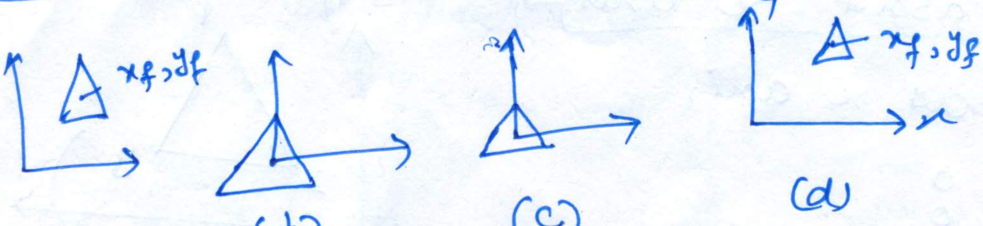
$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix} = ?$$

$$T(x_r, y_r) \cdot R(\theta) \cdot T(-x_r, -y_r) = R(x_r, y_r, \theta)$$

General fixed Point Scaling →

$$T(x_f, y_f) \cdot S(S_x, S_y) \cdot T(-x_f, -y_f) = S(x_f, y_f, S_x, S_y)$$

$$\boxed{T(-x_r, -y_r) = T^{-1}(x_r, y_r)}$$ inverse translation



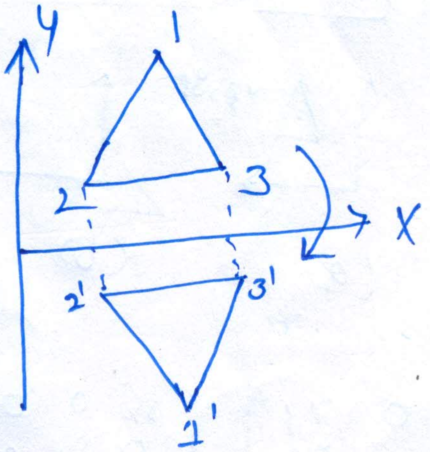
(a) (b) (c) (d)

$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection → ① It produces a mirror image of an object.

② The mirror image of an object for 2-D reflection is generated relative to an axis of reflection by rotating the object 180° about the reflection axis.

ⓐ Reflection about x axis →



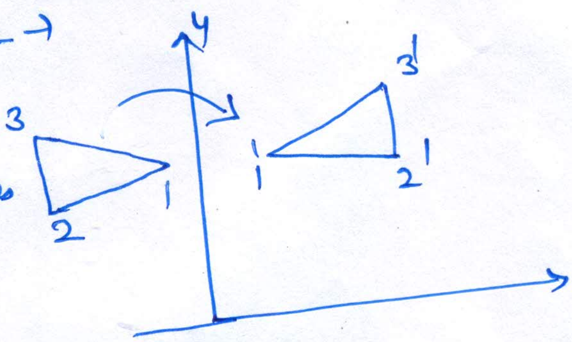
① $y = 0$ about x axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

② It keeps x values the same but flips the y values of co-ordinate positions.

③ To visualize the rotation transformation,
↳ Moving object out of xy plane
↳ Rotate ~~about~~ 180° abt x axis and place back into xy plane on the other side of x-axis

ⓑ Reflection about y axis →



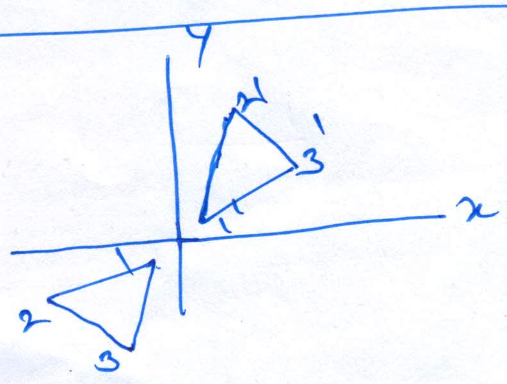
① $x = 0$ about y axis

② It keeps y co-ordinates values same

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

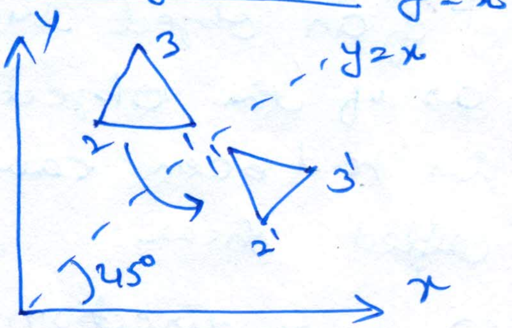
ⓒ Reflection relative to an axis perpendicular to xy plane

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(d) Reflection about Reflection axis as the diagonal line $y=x$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

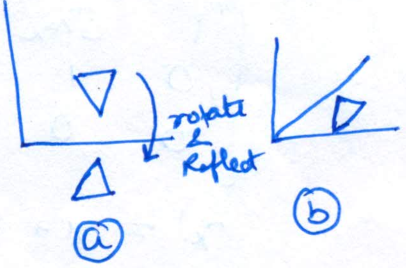


steps

(1) clockwise rotation about 45°

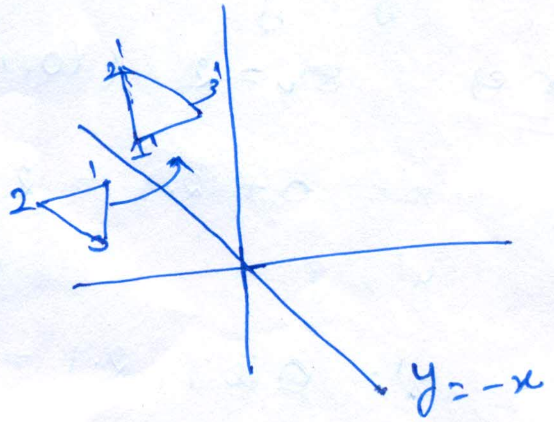
(2) reflect about x axis

(3) counter clockwise rotation



(e) Reflection about $y=-x$

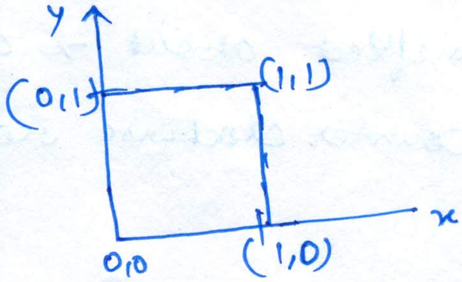
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Shear → A transformation that distorts the shape of an object such that transformed shape apears as if the object were composed of internal layer that had been caused to slide over each other is called shear.

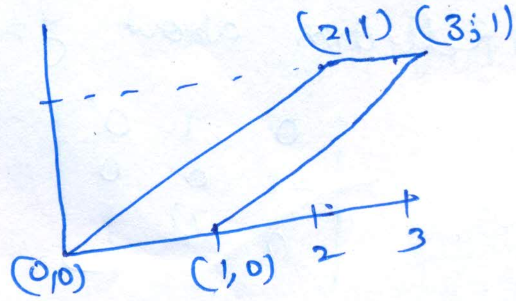
① An x-direction shear relative to x axis

$$\begin{bmatrix} 1 & Sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$x' = x + Sh_x \cdot y$

$y' = y$



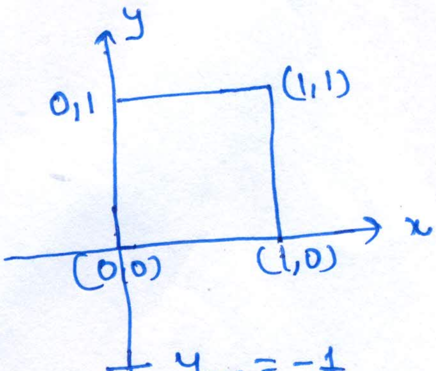
for eg $\quad Sh = 2 \quad ((0,1)$

$\left.\begin{array}{l} x' = 0 + 2.1 = 2 \\ y' = 1 \end{array}\right] \rightarrow (2,1)$

$\left.\begin{array}{l} x' = 1 + 2.1 = 3 \\ y' = 1 \end{array}\right] \rightarrow (3,1)$

② x-direction shears relative to other reference line

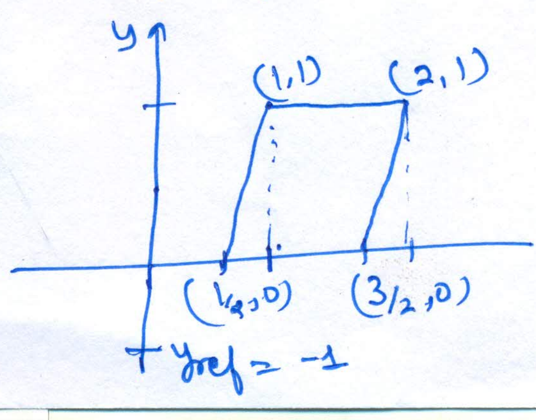
$$\begin{bmatrix} 1 & Sh_x & -Sh_x \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$x' = x + Sh_x(y - y_{ref})$,

$y' = y$

$Sh_x = \frac{1}{2}$, $y_{ref} = -1$



y-direction shear relative to line $x = x_{ref}$

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y \cdot x_{ref} \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = x$ & $y' = sh_y(x - x_{ref}) + y$

<u>for eg</u> $sh_y = 1/2$, $x_{ref} = -1$